

The graph of function f is shown on the right.

The graph consists of a diagonal line, an arc of a circle, then two additional diagonal lines.

SCORE: ____ / 4 PTS

[a] Evaluate $\int_{-10}^{10} f(x) dx$.

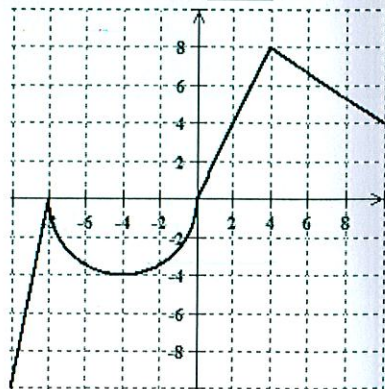
NOTE: You must show the arithmetic expression that you used to get your answer.

$$\begin{aligned} & \underbrace{-\frac{1}{2}(2 \times 10)} - \underbrace{\frac{1}{2}\pi(4)^2} + \underbrace{\frac{1}{2}(4)(8)} + \underbrace{\frac{1}{2}(6)(8+4)} \\ &= -10 - 8\pi + 16 + 36 \\ &= \underline{42 - 8\pi} \end{aligned}$$

$\left(\frac{1}{2}\right)$ POINT EACH
EXCEPT AS NOTED

[b] Evaluate $\int_{-4}^{10} f(x) dx$.

$$\begin{aligned} & \underbrace{-\int_{-4}^{10} f(x) dx}_{\textcircled{1}} = - \left[-\frac{1}{4}\pi(4)^2 + 16 + 36 \right] = \underline{4\pi - 52} \end{aligned}$$

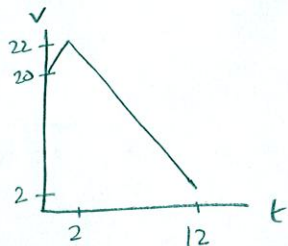


A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 20+t, & 0 \leq t \leq 2 \\ 26-2t, & 2 \leq t \leq 12 \end{cases}$.

SCORE: _____ / 5 PTS

- [a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds.

NOTE: You must show the arithmetic expression that you used to get your answer.

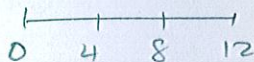


$$\begin{aligned} & \frac{1}{2}(2)(20+22) + \frac{1}{2}(10)(22+2) \\ & = 42 + 120 = 162 \text{ m} \end{aligned}$$

- [b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds using three subintervals and left endpoints.

NOTE: You must show the arithmetic expression that you used to get your answer.


$$\Delta x = \frac{12-0}{3} = 4$$



$$f(0)\Delta x + f(4)\Delta x + f(8)\Delta x = \underbrace{20 \cdot 4}_{\frac{1}{2}} + \underbrace{18 \cdot 4}_{\frac{1}{2}} + \underbrace{10 \cdot 4}_{\frac{1}{2}} = \underbrace{192}_{1} \text{ m}$$

Evaluate $\int_{-6}^0 (2\sqrt{36-x^2} - |x+2|) dx$ using the properties of definite integrals and interpreting in terms of area. SCORE: ____ / 5 PTS

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$$\textcircled{2} \quad 2 \int_{-6}^0 \sqrt{36-x^2} dx - \int_{-6}^0 |x+2| dx = 2 \cdot \frac{1}{4}\pi(6)^2 - \left(\frac{1}{2} \cdot 4 \cdot 4 + \frac{1}{2} \cdot 2 \cdot 2 \right)$$
$$= 18\pi - 10$$


Using the limit definition of the definite integral, and right endpoints, find $\int_{-4}^{-1} (2x^2 + 8x) dx$.

SCORE: ____ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{-1 - (-4)}{n} = \frac{3}{n}$$

$$\textcircled{1} \lim_{n \rightarrow \infty} \sum_{i=1}^n \underbrace{\left(2\left(-4 + \frac{3i}{n}\right)^2 + 8\left(-4 + \frac{3i}{n}\right) \right)}_{\textcircled{2}} \cdot \frac{3}{n} \textcircled{1}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(2\left(16 - \frac{24}{n}i + \frac{9}{n^2}i^2\right) - 32 + \frac{24}{n}i \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \underbrace{\left(-\frac{24}{n}i + \frac{18}{n^2}i^2 \right)}_{\textcircled{2}}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(-\frac{24}{n} \sum_{i=1}^n i + \frac{18}{n^2} \sum_{i=1}^n i^2 \right)$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \left(-\frac{24}{n} \frac{n(n+1)}{2} + \frac{18}{n^2} \frac{n(n+1)(2n+1)}{6} \right)$$

$$= 3(-12 + 6) \textcircled{1} \textcircled{1}$$

$$= -18 \textcircled{1}$$

+ $\textcircled{1}$ POINT IF YOU WROTE $\lim_{n \rightarrow \infty}$
ON EACH LINE THAT
CONTAINS "n"